Math 253
Spring 2021

Name: Hey

Quiz 6.2

1. Consider the function $f(x) = \dfrac{2}{(1-x)^3}$.

   (a) (6 points) Find the power series centered at 0 which converges to $f(x)$.

$$\frac{2}{(1-x)^3} = \frac{d}{dx}\left(\frac{1}{(1-x)^2}\right) = \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right)$$

$$= \frac{d^2}{dx^2}\left(\sum_{n=0}^{\infty} x^n\right) = \frac{d}{dx}\left(\sum_{n=1}^{\infty} n x^{n-1}\right)$$

$$= \sum_{n=2}^{\infty} n(n-1)\,x^{n-2}$$

   (b) (4 points) Give an interval of x-values for which this series converges. Justify why the series converges for those x-values. (There are several ways of doing this problem—pick your favorite!)

$\sum_{n=2}^{\infty} n(n-1)x^{n-2}$ has the same radius of convergence as $\sum_{n=1}^{\infty} n x^{n-1}$ which has the same radius of convergence as $\sum_{n=0}^{\infty} x^n$, which has $R = 1$. Hence, $\sum_{n=2}^{\infty} n(n-1)x^{n-2}$ converges on $(-1, 1)$